Steady-state analysis and design of activated sludge processes with a model including compressive settling

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Motivation for this work

Book: *Benchmarking of control strategies for wastewater treatment plants* by Gernaey, Jeppsson, Vanrolleghem and Copp (2015)

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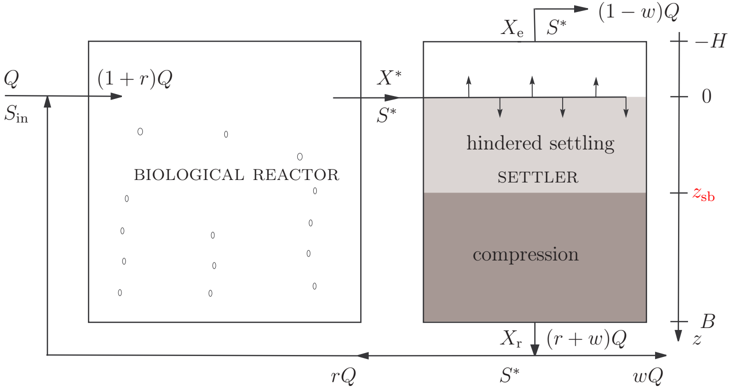
Previous publications: ideal point settler or hindered sedimentation

This work: include compressive settling

Reduced model of ASP in normal operation



Volumetric flow rate	Q	$[m^3/h]$	sludge blanket level z_{sb}
Soluble substrate	S	$[kg/m^3]$	
Particulate biomass	X	$[kg/m^3]$	
Recycle ratio	r	$[-]$	
Wastage ratio	w	$[-]$	

Equations for biological reactor

Completely stirred tank of volume $V = A_\mathrm{R} H_\mathrm{R}$

Standard mass balances:

$$V \frac{\mathrm{d}S^*}{\mathrm{d}t} = \underbrace{Q S_\mathrm{in} + r Q S_\mathrm{r}}_{\text{in}} - \underbrace{Q(1+r)S^*}_{\text{out}} - \underbrace{\frac{V \mu(S^*, X^*) X^*}{Y}}_{\text{consumed}}$$

$$V \frac{\mathrm{d}X^*}{\mathrm{d}t} = r Q X_\mathrm{r} - Q(1+r)X^* + V \big(\mu(S^*, X^*) - b\big) X^*$$

Biomass decay rate b

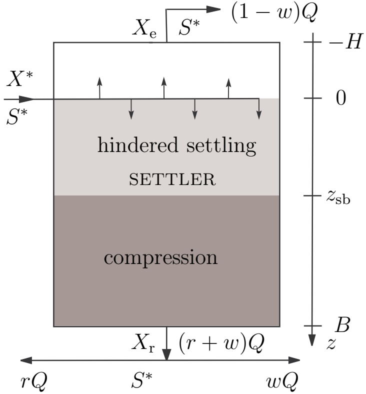
Growth kinetics (Monod, Contois, Haldane/Andrews, Webb):

$$\mu(S, X) = \hat{\mu} \frac{S(1 + \beta S / K_\mathrm{i})}{K_\mathrm{s} + K_\mathrm{C} X + S + S^2 / K_\mathrm{i}}$$

For design procedure (so far): $\mu(S)$ (Monod, Haldane/Andrews)

Equation for particulate concentration in settler

Focus on normal operation with $X_\mathrm{e} = 0$ and sludge blanket at z_sb



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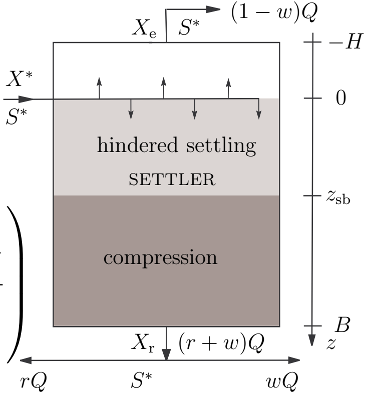
Bürger-Diehl PDE model for thickening zone:

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial z}\left(-\big(\underbrace{v_\mathrm{hs}(X)}_{\text{hindered settling}} + q\big)X + \underbrace{d(X)}_{\text{compression}} \frac{\partial X}{\partial z} \right)$$

bulk velocity $\quad q = \dfrac{(r+w)Q}{A_\mathrm{S}}$



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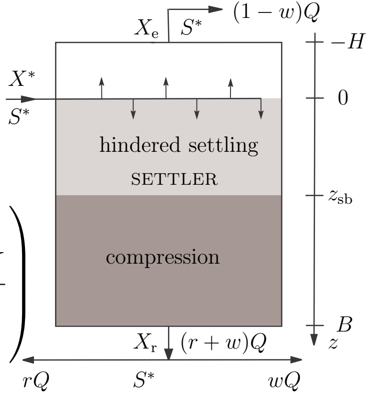

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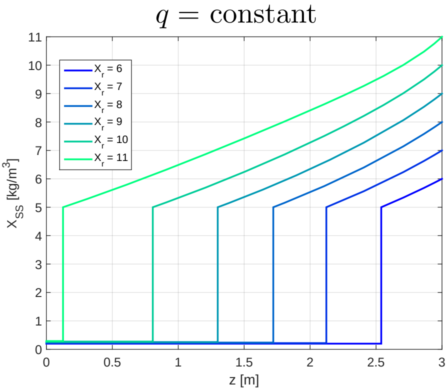
Note: Steady-state equation of PDE is ODE for $X_\mathrm{SS}(z)$, which is discontinuous (special mathematical theory)

Steady-state solutions $X_{\mathrm{SS}}(z)$ in thickening zone

Compression starts above the critical concentration $X_{\mathrm{c}} = 5$ kg/m^3



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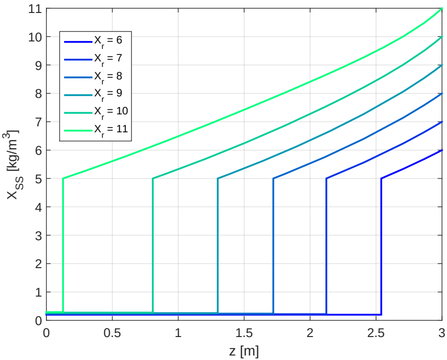
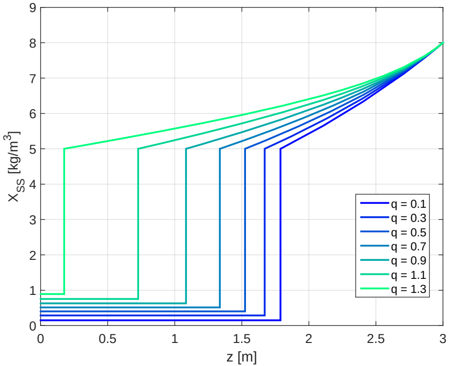


$X_{\mathrm{r}} = \text{constant}$



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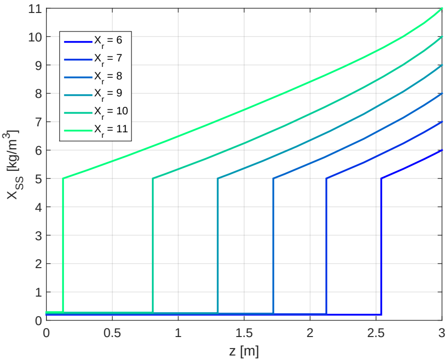

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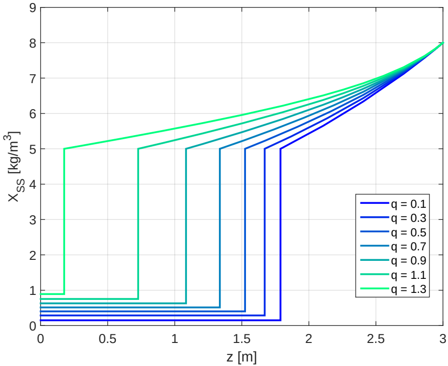

Observation: sludge blanket level z_{sb} depends on q and X_r

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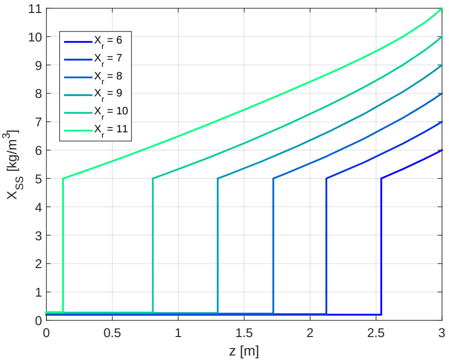

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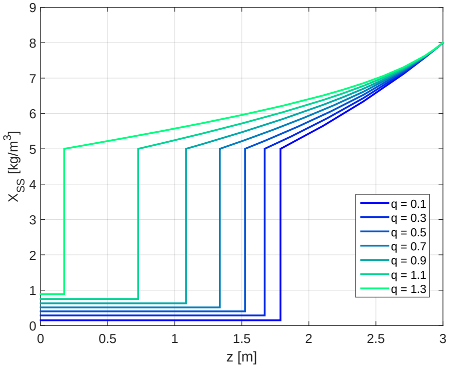

Observation: sludge blanket level z_{sb} depends on q and X_r

Result: capture this relation with algebraic equation replacing ODE

Steady-state equation for settler in normal operation

Result: For a given wanted sludge blanket level z_{sb}:

$$X_{\mathrm{r}} = U_{z_{\mathrm{sb}}}(q) := X_{z_{\mathrm{sb}}}^{\infty}\left(1 + \frac{\hat{q}_{z_{\mathrm{sb}}}}{q + \check{q}_{z_{\mathrm{sb}}}}\right), \qquad X_{z_{\mathrm{sb}}}^{\infty},\ \hat{q}_{z_{\mathrm{sb}}},\ \check{q}_{z_{\mathrm{sb}}} \text{ parameters}$$

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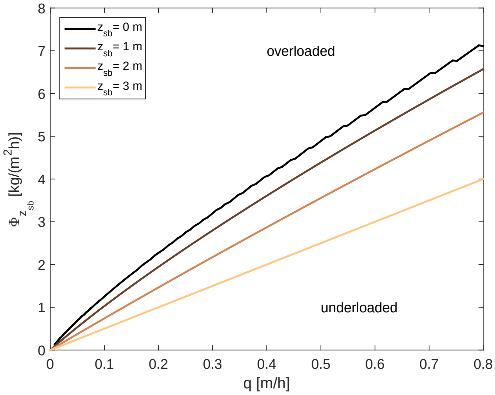
The flux capacity: $\Phi_{z_{\mathrm{sb}}}(q) := qU_{z_{\mathrm{sb}}}(q)$



Limiting flux because of compression!

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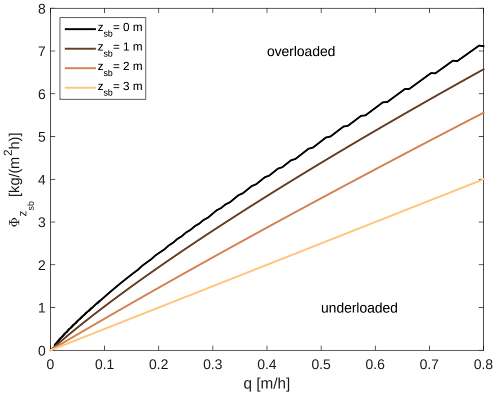

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Set of algebraic eqs for ASP in steady state

Solutions depend on r

For results and nice graphs, see paper

Design of ASP: total area

Solve equations for horizontal areas of reactor A_{R} and settler A_{S}:

$$A_{\mathrm{ASP}} = A_{\mathrm{R}} + A_{\mathrm{S}} = \frac{Qw(1+r)}{(r+w)H_{\mathrm{R}}\big(\mu(S^*_{\mathrm{ref}}) - b\big)} + \frac{Q(r+w)\big(w_{\max}(S_{\mathrm{in}}) - w\big)}{(\hat{q}_{z_{\mathrm{sb}}} + \check{q}_{z_{\mathrm{sb}}})\big(w - w_{\min}(S_{\mathrm{in}})\big)}$$

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- A_{ASP} proportional to Q
- A_{ASP} increases with S_{in}
- Small interval $w_{\min}(S_{\mathrm{in}}) < w < w_{\max}(S_{\mathrm{in}})$
- $w_{\min}$ corresponds to $X_{\mathrm{r,max}}$ and vice versa

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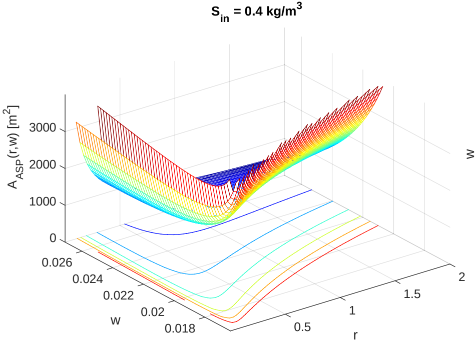
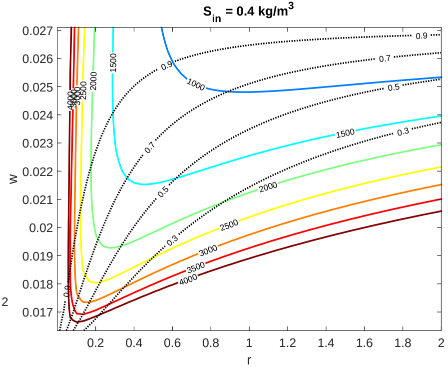
Study $A_{\mathrm{ASP}}(r, w)$

Design of ASP: total area given $z_{\rm sb}$, Q, $S_{\rm in}$, $S^*_{\rm ref}$

Graph and contours of $A_{\rm ASP} = A_{\rm ASP}(r, w)$





Dotted black curves in right plot show ratios $A_{\rm R}/A_{\rm ASP}$

One diagram: Decide $A_{\rm R}$, $A_{\rm S}$ and nominal operating point (r, w)

Main conclusions

- New algebraic equation $X_{\mathrm{r}} = U_{z_{\mathrm{sb}}}(q)$ means that flux capacity due to compressive settling easily included in analysis
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Related work: See our poster on a plug-flow reactor + settler

Thank you for your attention!