Steady-state analysis and design of activated sludge processes with a model including compressive settling

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Motivation for this work


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**Reduced steady-state model** with only few equations can be preferable, e.g., in the first design considerations (plant area)
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**Reduced steady-state model** with only few equations can be preferable, e.g., in the first design considerations (plant area)

**Previous publications:** ideal point settler or hindered sedimentation

**This work:** include compressive settling
Reduced model of ASP in normal operation

Volumetric flow rate \( Q \) [m\(^3\)/h]
Soluble substrate \( S \) [kg/m\(^3\)]
Particulate biomass \( X \) [kg/m\(^3\)]
Recycle ratio \( r \) [-]
Wastage ratio \( w \) [-]

sludge blanket level \( z_{sb} \)
Equations for biological reactor

Completely stirred tank of volume $V = A_R H_R$

Standard mass balances:

\[
V \frac{dS^*}{dt} = Q S_{\text{in}} + r Q S_r - Q (1 + r) S^* - V \frac{\mu(S^*, X^*) X^*}{Y} \] 
consumed

\[
V \frac{dX^*}{dt} = r Q X_r - Q (1 + r) X^* + V (\mu(S^*, X^*) - b) X^* \]

Biomass decay rate $b$

Growth kinetics (Monod, Contois, Haldane/Andrews, Webb):

\[
\mu(S, X) = \hat{\mu} \frac{S(1 + \beta S/K_i)}{K_s + K_C X + S + S^2/K_i}
\]

For design procedure (so far): $\mu(S)$ (Monod, Haldane/Andrews)
Focus on normal operation with $X_e = 0$ and sludge blanket at $z_{sb}$
Equation for particulate concentration in settler

Focus on normal operation with $X_e = 0$ and sludge blanket at $z_{sb}$

Bürger-Diehl PDE model for thickening zone:

$$\frac{\partial X}{\partial t} = \frac{\partial}{\partial z} \left( - \left( v_{hs}(X) + q \right) X + d(X) \frac{\partial X}{\partial z} \right)$$

bulk velocity $q = \frac{(r + w)Q}{A_S}$
Equation for particulate concentration in settler

Focus on normal operation with \( X_e = 0 \) and sludge blanket at \( z_{sb} \)

Bürger-Diehl PDE model for thickening zone:

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bulk velocity \( q = \frac{(r + w)Q}{A_S} \)

**Note:** Steady-state equation of PDE is ODE for \( X_{SS}(z) \), which is discontinuous (special mathematical theory)
Steady-state solutions $X_{SS}(z)$ in thickening zone

Compression starts above the critical concentration $X_c = 5 \text{ kg/m}^3$

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$X_r = \text{constant}$
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$q = \text{constant}$

$X_r = \text{constant}$

Observation: sludge blanket level $z_{sb}$ depends on $q$ and $X_r$
Steady-state solutions $X_{SS}(z)$ in thickening zone

Compression starts above the critical concentration $X_c = 5 \text{ kg/m}^3$

$q = \text{constant}$

$q = 0.1$
$q = 0.3$
$q = 0.5$
$q = 0.7$
$q = 0.9$
$q = 1.1$
$q = 1.3$

$X_r = \text{constant}$

$X_r = 6$
$X_r = 7$
$X_r = 8$
$X_r = 9$
$X_r = 10$
$X_r = 11$

Observation: sludge blanket level $z_{sb}$ depends on $q$ and $X_r$

Result: capture this relation with algebraic equation replacing ODE
Result: For a given wanted sludge blanket level $z_{sb}$:

$$X_r = U_{z_{sb}}(q) := X_{z_{sb}}^\infty \left(1 + \frac{\hat{q}_{z_{sb}}}{q + \tilde{q}_{z_{sb}}}\right), \quad X_{z_{sb}}^\infty, \hat{q}_{z_{sb}}, \tilde{q}_{z_{sb}} \text{ parameters}.$$
Steady-state equation for settler in normal operation

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The flux capacity:  

$$\Phi_{z_{sb}}(q) := q U_{z_{sb}}(q)$$

**Limiting flux because of compression!**
Steady-state equation for settler in normal operation

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Set of algebraic eqs for ASP in steady state

Solutions depend on $r$

For results and nice graphs, see paper
Design of ASP: total area

Solve equations for horizontal areas of reactor $A_R$ and settler $A_S$:

$$A_{ASP} = A_R + A_S = \frac{Qw(1 + r)}{(r + w)H_R(\mu(S^*_{ref}) - b)} + \frac{Q(r + w)(w_{max}(S_{in}) - w)}{(\hat{q}_{zs} + \tilde{q}_{zs})(w - w_{min}(S_{in}))}$$

$$r > 0, \quad w_{min}(S_{in}) < w < w_{max}(S_{in})$$
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$$r > 0, \quad w_{\text{min}}(S_{\text{in}}) < w < w_{\text{max}}(S_{\text{in}})$$

- $A_{ASP}$ proportional to $Q$
- $A_{ASP}$ increases with $S_{\text{in}}$
- Small interval $w_{\text{min}}(S_{\text{in}}) < w < w_{\text{max}}(S_{\text{in}})$
- $w_{\text{min}}$ corresponds to $X_{r,\text{max}}$ and vice versa
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Choose wanted $z_{sb}$, $Q$, $S_{in}$, and $S^*_{ref}$
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Choose wanted $z_{sb}$, $Q$, $S_{in}$, and $S_{ref}^*$

Study $A_{ASP}(r, w)$
Design of ASP: total area given $z_{sb}$, $Q$, $S_{in}$, $S_{ref}$

Graph and contours of $A_{ASP} = A_{ASP}(r, w)$

Dotted black curves in right plot show ratios $A_R/A_{ASP}$

One diagram: Decide $A_R$, $A_S$ and nominal operating point $(r, w)$
Main conclusions

- **New algebraic equation** $X_r = U_{zs_b}(q)$ means that flux capacity due to compressive settling easily included in analysis
- **Design procedure**: explicit formulas — one diagram
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- **Design procedure:** explicit formulas — one diagram

Related work: See our poster on a plug-flow reactor + settler

Thank you for your attention!