Critical analysis of constitutive functions for hindered settling velocity in 1-D settler models

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BIOMATH
Model-based analysis and optimization of bioprocesses

DEPARTMENT OF MATHEMATICAL MODELLING, STATISTICS AND BIOINFORMATICS
What’s in a name...

Discrete settling
- Low concentrations
- Particles settle as individuals

Hindered settling
- Intermediate concentrations
- Particles settle as one mass

Compression settling
- High concentrations
- Particles form network with compressive stress
What do we want?

Simulate SST dynamics under dry and wet-weather conditions

Compression settling important to simulate wet-weather dynamics
Framework available: Bürger-Diehl model

\[ \frac{\partial X}{\partial z} = -\frac{\partial}{\partial z} (v_c(t)X) \]

\[ -\frac{\partial}{\partial z} (v_{hs}(X) X) \]

\[ + \frac{\partial}{\partial z} \left( v_{hs}(X) \frac{\rho_s}{(\rho_s - \rho_f)g} \frac{d\sigma_e(X)}{dX} \frac{\partial X}{\partial z} \right) \]

\[ + \frac{Q_f(t)X_f(t)}{A} \delta(z) \]

Model selection and calibration of constitutive functions

1. Hindered settling: \( v_{hs}(X) \)
2. Solids stress: \( \sigma_e(X) \)
Selection of hindered settling functions

Exponential

\[
v_{hs}(X) = V_0 e^{-r_v X}
\]

\[
v_{hs}(X) = V_0 \left( e^{-r_H X} - e^{-r_P X} \right)
\]

Power-law

\[
v_{hs}(X) = k X^{-n}
\]

\[
v_{hs}(X) = \frac{V_0}{1 + \left( \frac{X}{\bar{X}} \right)^q}
\]

Vesilind (1968)

Takács et al. (1991)

Cole (1968)

Diehl (2014)
Calibration

Dataset 1
De Clercq et al. (2005)

Dataset 2
Locatelli et al. (2014)
Calibration

Dataset 1
De Clercq et al. (2005)

Dataset 2
Locatelli et al. (2014)
45 min 1-D simulation exponential $v_{hs}$

$\nu_s = v_{hs}(X) \left( 1 - \frac{\rho_s \sigma_e(X) \frac{\partial X}{\partial z}}{\rho_s - \rho_f} gX \right)$ for $X > X_{crit}$

SBH underpredicted
$v_s$ overpredicted
Compression needed
6h 1-D simulation hindered settling

SBH \downarrow
Settling velocity \uparrow
\Rightarrow v_{hs} > v_{comp}

SBH \uparrow
Settling velocity \downarrow
\Rightarrow v_{hs} - v_{comp}
6h 1-D simulation hindered settling

Exponential functions
- Contradictory behaviour
- Not to be combined with compression function

Power-law functions
- Behave as expected
- Can be combined with compression function
Measured velocity profiles

Velocity profile

Data

Sludge Blanket Height
1-D simulation with $v_{hs}$ Vesilind

Velocity profile

Sludge Blanket Height
Conclusions

» Exponential hindered settling functions
  › Partially lump in compression behaviour
  › Cannot be combined with compression function
    → use for simple 1-D models

» Power-law hindered settling functions
  › Represent ‘ideal’ thickening
  › Can be combined with compression function
    → use for advanced 1-D models
model-based analysis and optimization of bioprocesses

Kinetic model
Modelling of process dynamics

Impact of spatial heterogeneity on process kinetics

Impact of distributed versus lumped kinetics

Computational Fluid Dynamics (CFD)
Modelling hydrodynamics

Population Balance Model (PBM)
Modelling of distributed population properties

Impact of flow properties on distributed property of a population
What do we know?
Comparison velocity profiles at $t=20h$

<table>
<thead>
<tr>
<th></th>
<th>Exp. data</th>
<th>1-D simulations</th>
<th>1-D simulations</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Locatelli et al. [2014]</td>
<td>$v_{hs}$ of Vesilind</td>
<td>$v_{hs}$ of Diehl</td>
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<tr>
<td>SBH [m]</td>
<td>0.15</td>
<td>0.21</td>
<td>0.02</td>
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<tr>
<td>$v_{s,\text{bottom}}$ [m/s]</td>
<td>0.0124</td>
<td>0.0022</td>
<td>0.0059</td>
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<tr>
<td>$v_{s,\text{SBH}}$ [m/s]</td>
<td>0.0380</td>
<td>0.0027</td>
<td>0.0200</td>
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<tr>
<td>$X_{\text{bottom}}$ [g/l]</td>
<td>32.0</td>
<td>21.8</td>
<td>315.6</td>
</tr>
<tr>
<td>$X_{\text{SBH}}$ [g/l]</td>
<td>9.0</td>
<td>10.5</td>
<td>31.6</td>
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