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Analyses of Activated Sludge Processes consisting of a Plug-Flow Reactor and a Non-ideal Settler

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Summary of key findings

An activated sludge process (ASP) consisting of a plug-flow reactor (PFR) and a non-ideal settler is modelled and analysed. One soluble substrate component and one particulate biomass are assumed. The biomass growth rate is described by a Monod function. The settler model includes hindered settling and compression. A model describing the steady-state behaviour of the ASP is derived which constrains the settler to work with a fixed sludge blanket height in the thickening zone. The model provides new understanding for these types of ASPs and may be used for novel design schemes. The numerical example suggests that the steady-state solutions of the ASP give a one-parameter family of solutions, where the parameter is the recycle ratio r.

Background and relevance

Steady-state modelling and analysis of an ASP have been studied during decades despite the fact that a wastewater treatment plant may never be in steady state. The reasons include that a steady-state analysis of a dynamic model can give additional insights compared to only study a more complex model numerically, it can provide simplified design rules, and can provide feasible initial values for numerical optimization of a dynamic process model. In this study we will analyse an ASP consisting of a plug-flow reactor (PFR) and a non-ideal settler. Previous analyses of ASPs with a PFR configuration can be found in, for example, San (1989) and Muslu (2000). Often very simplified assumptions such as the settler works in normal operation regardless of the loading conditions are used. In our work, we adopt a steady state settler model from Diehl et al. (2015) where hindered settling and compression at high sludge concentrations are taken into account.

Methods

We consider an ASP according to Figure 1. The PFR has a constant vertical cross-sectional area A_{R} and length h, so the volume is $V = A_{\mathrm{R}}h$. Let x denote the variable of the horizontal axis in the PFR from the inlet ($x = 0$) to the outlet ($x = h$). The settler has a constant cross-sectional area A_{S}. We assume two constituents, namely one particulate biomass and one growth limiting dissolved substrate. The influent volumetric flow rate and substrate concentration are denoted by Q and S_{in}, respectively. It is assumed that no biomass is present in the influent ($X_{\mathrm{in}} = 0$).

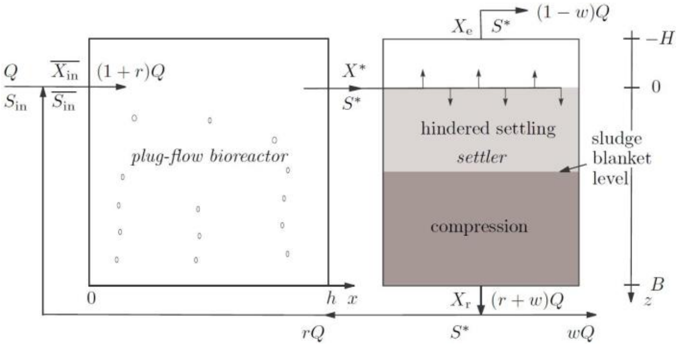

Figure 1. The activated sludge process consisting of a plug-flow reactor (PFR) and a settler. The steady-state variables are shown as well as the horizontal x-axis of the PFR and the vertical z-axis of the settler.

The input concentrations to the PFR are denoted by $\overline{S_{\rm in}}$ and $\overline{X_{\rm in}}$, and the outputs by S^* and X^*. It is assumed that there are no reactions in the settler, so that only the particulate biomass is influenced. The substrate concentration is thus unchanged equal to S^* through the settler. The effluent at the top is $X_{\rm e}$ and the recycle sludge concentration is $X_{\rm r}$. The recycle flow rate is rQ and the waste flow rate wQ where $r \geq 0$ and $0 \leq w \leq 1$. The growth kinetics in the PFR are described by the Monod function

$$\mu(S) = \mu_{\max}\frac{S}{K_{\rm s} + S}, \qquad (1)$$

where $\mu_{\max}$ is the maximum specific growth rate and $K_{\rm s}$ is the half-saturation constant.

The processes in the settler are described by a steady state approximation of a PDE which describes a hindered settling velocity function and a compression function (Bürger et al., 2011). The behaviour of a real settler can be divided into three qualitatively different operations: underloaded, overloaded and normal operation. By normal operation we mean that all the biomass fed to the settler is conveyed through the thickening zone and that there exists a sludge blanket in the thickening zone. In this study we are only interested in steady-state solutions under normal operation and therefore set $X_{\rm e} = 0$.

Results and Discussions

The three mass balances in steady-state with $X_{\rm e} = 0$ are:

$$Q(1+r)\overline{S_{\rm in}} = QS_{\rm in} + rQS^*, \qquad (2)$$

$$Q(1+r)\overline{X_{\rm in}} = rQX_{\rm r}, \qquad (3)$$

$$Q(1+r)X^* = Q(r+w)X_{\rm r}. \qquad (4)$$

For constant values on r and w, these three equations contain 5 variables: $\overline{S_{\rm in}}$, S^*, $\overline{X_{\rm in}}$, X^*, and $X_{\rm r}$. These mass balances should be complemented with equations relating the input and output variables of the PFR and settler. For the settler, we shall utilize the results of Diehl et al. (2015) where it is shown that the following simple relationship is a reasonable steady state approximation of a settler constrained to have a fixed sludge blanket height in the thickening zone:

$$X_{\rm r} = U_{z_{\rm sb}}(q) := X^{\infty}_{z_{\rm sb}}\left(1 + \frac{\hat{q}_{z_{\rm sb}}}{q + \check{q}_{z_{\rm sb}}}\right), \quad \text{where} \quad q := \frac{Q(r+w)}{A_{\rm S}} \qquad (5)$$

where $X^{\infty}_{z_{\rm sb}}$, $\hat{q}_{z_{\rm sb}}$ and $\check{q}_{z_{\rm sb}}$ are parameters which depend on the chosen sludge blanket level $z_{\rm sb}$ (see the numerical example).

Let $S(x)$ and $X(x)$ denote the concentrations at location x in the PFR, see Figure 1. Then the following hold: $X(0) = \overline{X_{\rm in}}$, $S(0) = \overline{S_{\rm in}}$, $X(h) = X^*$, $S(h) = S^*$. Applying conservation of mass in the PFR gives:

$$\frac{Q(1+r)}{A_{\rm R}}\frac{dS}{dx} = -\mu[S(x)]\frac{X(x)}{Y}, \qquad (6)$$

$$\frac{Q(1+r)}{A_{\rm R}}\frac{dX}{dx} = \mu[S(x)]X(x). \qquad (7)$$

Note that (6) and (7), together with the boundary values, give:

$$\frac{Q(1+r)}{A_{\rm R}}\frac{d(YS+X)}{dx} = 0 \quad \Longrightarrow \quad Y\overline{S_{\rm in}} + \overline{X_{\rm in}} = YS(x) + X(x) = YS^* + X^*. \qquad (8)$$

Replacing $X(x)$ from (8) in (6) and integrating, we get the following expression in the PFR:

$$-\frac{YQ(1+r)}{A_{\rm R}}\int_{\overline{S_{\rm in}}}^{S^*}\frac{d\sigma}{\mu(\sigma)[Y(\overline{S_{\rm in}} - \sigma) + \overline{X_{\rm in}}]} = h. \qquad (9)$$

Equations (8)–(9) should be complemented with the mass balances (2)–(4) and (5) for the settler. It is easy to show that the unknowns $\overline{S_{\rm in}}, \overline{X_{\rm in}}, S^*, X^*, X_{\rm r}, r$ and w satisfy:

$$\overline{S_{\rm in}} = S_{\rm in} - \frac{rw}{(1+r)Y} U_{z_{\rm sb}}(q), \tag{10}$$

$$\overline{X_{\rm in}} = \frac{r}{(1+r)} U_{z_{\rm sb}}(q), \tag{11}$$

$$S^* = S_{\rm in} - \frac{w}{Y} U_{z_{\rm sb}}(q), \tag{12}$$

$$X^* = \frac{r+w}{1+r} U_{z_{\rm sb}}(q), \tag{13}$$

$$X_{\rm r} = U_{z_{\rm sb}}(q). \tag{14}$$

The integral in (9) can be evaluated using the same technique as in Zambrano et al. (2015):

$$\frac{V\mu_{\max}}{(1+r)Q}(YS^* + X^*) = (YS^* + X^* + YK_{\rm s}) \ln\left(\frac{X^*}{\overline{X_{\rm in}}}\right) + YK_{\rm s} \ln\left(\frac{\overline{S_{\rm in}}}{S^*}\right), \tag{15}$$

which can be written in a more compact form as:

$$a(r,w) = b(r,w) \ln\big(c(r,w)\big) + YK_{\rm s} \ln\big(d(r,w)\big), \tag{16}$$

where

$$a(r,w) \coloneqq \frac{V\mu_{\max}}{(1+r)Q}(YS^* + X^*) = \frac{V\mu_{\max}}{(1+r)Q}\left(YS_{\rm in} + \frac{r(1-w)}{(1+r)} U_{z_{\rm sb}}(q)\right),$$

$$b(r,w) \coloneqq YS^* + X^* + YK_{\rm s} = Y(S_{\rm in} + K_{\rm s}) + \frac{r(1-w)}{(1+r)} U_{z_{\rm sb}}(q),$$

$$c(r,w) \coloneqq \frac{X^*}{\overline{X_{\rm in}}} = 1 + \frac{w}{r},$$

$$d(r,w) \coloneqq \frac{\overline{S_{\rm in}}}{S^*} = \frac{(1+r)YS_{\rm in} - rwU_{z_{\rm sb}}(q)}{(1+r)\left(YS_{\rm in} - wU_{z_{\rm sb}}(q)\right)}.$$

Straightforward calculations give the following expression for the sludge age:

$$\theta = \frac{A_{\rm R}\int_0^h X(x)dx}{wQX_{\rm r}} = \frac{1}{\mu_{\max}}\left[1 + \frac{(1+r)YK_{\rm s}}{wU_{z_{\rm sb}}(q)} \ln\big(d(r,w)\big)\right] =: \theta(r,w), \qquad (w > 0). \tag{17}$$

It can be shown that if the parameter values are such that (16) implicitly defines a smooth function $w = w(r)$, which satisfies $0 < w(r) < 1$ for $r > 0$, then the sludge age satisfies:

$$\lim_{r\to 0} \theta\big(r, w(r)\big) = \frac{1}{\mu(S_{\rm in})}. \tag{18}$$

We conjecture that this indeed is the minimum sludge age, see the numerical example. Numerical examples with physically realistic parameter values indicate that there exists a relation $w = w(r)$ implicitly defined by (16) and satisfies $0 < w(r) < 1$ for $r > 0$; see the next section.

Remark. If we assume ideal settling so that (4) holds without imposing (5) we get the following result

$$V = \bar{a}\left[\frac{K_{\rm s}}{(S_{\rm in} + \bar{b}S^*)} \ln\left(\frac{\bar{d}S^*(1+r)}{S_{\rm in} + rS^*}\right) + \bar{c}\ln(\bar{d})\right], \tag{19}$$

where $\bar{a} = -\frac{Q(1+r)^2 w}{\mu_{\max}(r+w)}$; $\bar{b} = \frac{r(w-1)}{r+w}$; $\bar{c} = \frac{r+w}{w(1+r)}$; $\bar{d} = \frac{r}{r+w}$. This explicit expression for the PFR volume is to the best of the authors' knowledge new.

Numerical example

For the PFR the following constants and parameters were used: $V = 3000\ \mathrm{m}^3$, $\mu_{\max} = 0.17\ \mathrm{h}^{-1}$, $K_\mathrm{s} = 0.05\ \mathrm{kg/m}^3$, $Y = 0.7$. For the settler we let $A_\mathrm{S} = 1500\ \mathrm{m}^2$, $B = 3\ \mathrm{m}$, $z_\mathrm{sb} = 1\ \mathrm{m}$, $X_{z_\mathrm{sb}}^\infty = 6.52\ \mathrm{kg/m}^3$, $\hat{q}_{z_\mathrm{sb}} = 0.32\ \mathrm{m/h}$ and $\check{q}_{z_\mathrm{sb}} = 0.44\ \mathrm{m/h}$. Standard parameters for the hindered settling and compression functions were used, see details in Diehl et al. (2015). In Figure 2, we keep the influent volumetric flow rate constant $Q = 1000\ \mathrm{m}^3/\mathrm{h}$ and plot the one-parameter family of solutions for some values of the influent substrate concentration.

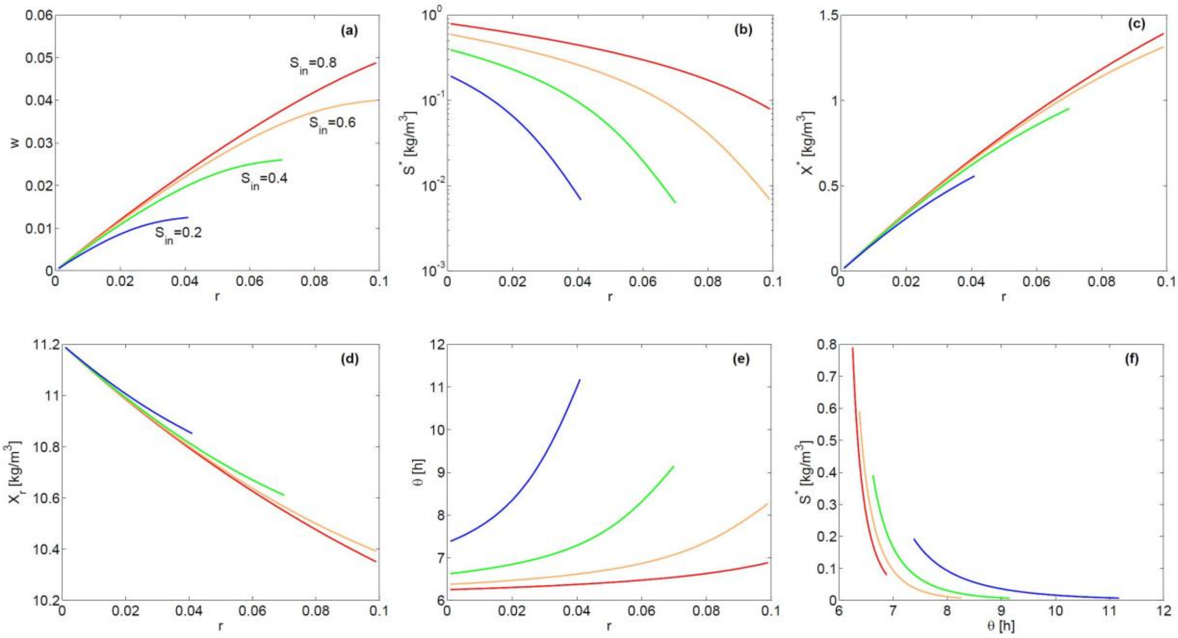

Figure 2. Steady state solutions of an ASP with a PFR and a settler constrained to have a fixed sludge blanket level. Solutions for 4 different influent substrate concentrations S_{in} [kg/m^3] are shown.

This numerical example (as well as others not included here) shows a unique solution w for every fixed $r > 0$. In Figure 2a it is shown that the range of possible solutions for $w = w(r)$ increases as S_{in} increases. In Figures 2b-2e it is seen that S^* and X_r decrease with increasing r, whereas X^* and θ increase. In Figure 2f it is shown that (as expected) the effluent substrate S^* decreases as the sludge age increases. But it is also seen that for a given sludge age, the effluent substrate S^* decreases as S_{in} increases. Hence, in this case the sludge age does not uniquely determine the effluent substrate which is in contrast to the classical text box example where the ASP only has one continuous stirred reactor. We conjectured above that (18) is the minimum sludge age. This is supported in this numerical example where the minimum sludge age decreases as S_{in} increases (see Figures 2e and 2f).

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