Critical analysis of constitutive functions for hindered settling velocity in 1-D settler models

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Summary of key findings

Advanced 1-D models for secondary clarifiers aim to incorporate more realism by explicitly accounting for several phenomena that influence the settling process. For each of them a valid mathematical expression needs to be selected to obtain a calibrated model that can be used for operation and control. The presented work evaluates different available expressions for hindered settling based on long term batch settling data. The analysis shows that the exponential forms which are most commonly used in traditional SST models not only account for hindered settling but partly lump other phenomena as well. This makes them unsuitable for advanced 1-D models that explicitly include each phenomenon in a modular way. A power-law function is shown to be more appropriate to describe the hindered settling velocity.

Background and relevance

A new 1-D model which allows improved and more realistic simulation of secondary clarifiers has recently been presented (Bürger et al., 2011, 2012). All implementation details can be found in (Bürger et al., 2013). This new model includes a numerical scheme for the solution of a governing partial differential equation (PDE - provided below) that is consistent with current PDE theory. Moreover it allows the modeler to account for several phenomena (such as hindered settling, sludge compression and inlet dispersion) in a modular way making it very flexible in its application. *Thus the constitutive expressions describing the different phenomena (v_{hs}, σ_e, d_{disp} - indicated in red) can easily be updated or replaced whenever future research provides further insight in any of these phenomena.*

$$
\begin{aligned}
\frac{\partial X}{\partial t} = \\
& -\frac{\partial}{\partial z}\left(v_c(t)\,X + \mathbf{v_{hs}(X)}\,X\right) && \text{convective flow and gravitary settling} \\
& +\frac{\partial}{\partial z}\left(\mathbf{v_{hs}(X)}\frac{\rho_s}{\left(\rho_s-\rho_f\right)g}\frac{\mathrm{d}\boldsymbol{\sigma_e}\mathbf{(X)}}{\mathrm{d}X}\frac{\partial X}{\partial z}\right) && \text{compression settling} \\
& +\frac{\partial}{\partial z}\left(\mathbf{d_{disp}(z, Q_f(t))}\frac{\partial X}{\partial z}\right) && \text{inlet dispersion} \\
& +\frac{Q_f(t)X_f(t)}{A}\delta(z) && \text{incoming feed flow}
\end{aligned}
$$

In order to obtain a 1-D model that can be used for operation and control, it is imperative to select valid constitutive functions and to develop a calibration protocol for their parameters. For each of these constitutive functions several alternatives have been presented in literature (Richardson and Zaki, 1954; Vesilind, 1968; Takács et al., 1991; Cho et al., 1993; De Clercq et al., 2008; Diehl, 2014; Ramin et al., 2014). Since gravitational settling and compression settling are the governing processes to predict important operating variables such as sludge blanket height and recycle concentration and given that both settling regimes depend on a constitutive function for the hindered settling velocity (v_{hs}) it follows that the hindered settling velocity should be the first function to examine. An inadequate choice for the hindered settling function could after all impede the further selection and calibration process.

Literature on hindered settling is dominated by exponential and power law functions with the exponential functions being most established in commonly used layer models. However, since hindered settling is the only driving force considered in many of these layer models, the exponential relation may not have been selected for its superiority in describing the phenomenon of hindered settling as such but for its overall performance including the ability to partially compensate for missing phenomena. De Clercq et al. (2008) found that an exponential expression for the hindered settling velocity in combination with an expression for σ_e was not able to describe experimental batch settling data whereas a power-law function could. Moving to advanced settler models which try to explicitly account for each phenomenon separately thus requires to re-evaluate available hindered settling functions. This work evaluates the validity of different constitutive functions for hindered settling based on detailed data of long-term batch experiments.

Results and discussion

Three hindered settling functions were selected for analysis: the most commonly used exponential functions of Vesilind (1986) (eq. 1) and Takács et al. (1991) (eq. 2) and a power-law function proposed by Diehl (2014) (eq. 3). The latter selected this power-law function based on an extensive study to identify an expression for v_{hs} by solving an inverse problem. In these equations X represents the sludge concentration and V_0, r_V, r_H, r_P, $\overline{X}$ and q are positive parameters to be calibrated.

$$v_{hs}(X) = V_0\, e^{-r_V X} \qquad (1)$$

$$v_{hs}(X) = V_0 \left(e^{-r_H X} - e^{-r_P X}\right) \qquad (2)$$

$$v_{hs}(X) = \frac{V_0}{1 + \left(\frac{X}{\bar{X}}\right)^q} \qquad (3)$$

The analysis of the constitutive functions was performed based on two sets of batch settling data. The first dataset was collected by De Clercq et al. (2005) who performed in depth-batch experiments by means of a radio-tracer. These experiments were performed with sludge from the WWTP of Destelbergen (Belgium) at three different initial concentrations (2.4, 3.23 and 4.3 g/l) resulting in three sets of complete concentration profiles during 6 hours of settling.

The second set of data was collected by Locatelli et al. (2014) who developed an experimental procedure to measure settling velocities within the sludge blanket without disturbing it. The measurements were performed by an ultrasonic transducer which was installed above a settling column to perform vertical measurements of the settling velocity. These experiments were performed with sludge from the WWTP of Rosheim (France) at 7 different initial concentrations (1.5, 2.7, 3.2, 3.8, 4.0, 4.6 and 5.6 g/l) resulting in 7 sets of complete velocity profiles during 1-22 hours (depending on the experiment) of settling.

For each dataset the settling parameters of eqs. 1-3 were calibrated. The calibration protocol was as follows: for each initial batch experiment, the slope of the linear descend of the sludge blanket was calculated providing the hindered settling velocity at the initial concentration of the batch. This resulted in 3 datapoints for the first dataset and 6 datapoints for the second dataset. Subsequently, the parameters for the hindered settling functions were calibrated by solving a least squares problem. The results are shown in Figure 1. (Identifiability of the settling parameters for the different functions was also investigated but these results are beyond the scope of the presented abstract.)

From Figure 1 it can be observed that all 3 hindered settling functions provide a good fit to the data. Quantifying the fit with a criterium for model selection such as Akaike's information criterium (AIC - values not shown) also indicated all functions performed equally well.

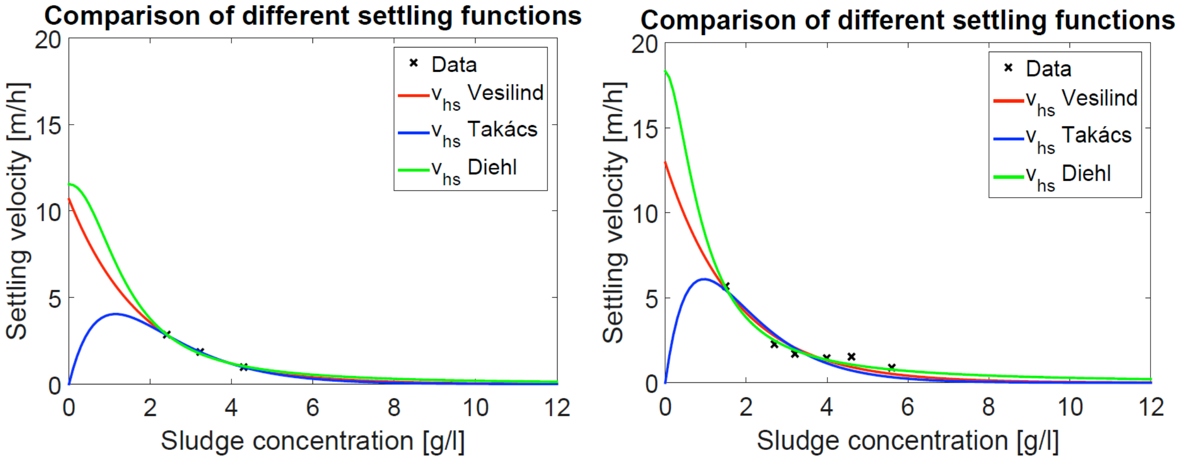

Figure 1: Comparison of different hindered settling functions against measured data of De Clercq et al. (2005) (left) and data of Locatelli et al. (2014) (right).

However, a similar fit to the hindered settling data does not imply similar simulation results when implementing the functions in a 1-D model. Both at low and high concentration ranges (where hindered settling cannot be measured) differences between the presented functions can influence the predictions of the 1-D model. At low concentrations (<1 g/l) discrete settling prevails (i.e. settling of individual flocs with size as the dominant influence rather than concentration). Including discrete settling in a 1-D model remains challenging and is outside of the scope of this work. At higher concentrations (> approx. 6 g/l) hindered settling will be slowed down by the formation of a network of particles that undergo a compressive force. The choice of the hindered settling function in this region will thus be important for the subsequent selection and calibration of a constitutive function for the solids stress σ_e.

Each of the hindered settling functions were implemented in the Bürger-Diehl settler model and used to simulate the experimental batch settling data. For the first dataset, the resulting predictions of SBH are provided in Figure 2. Only the results for eq. 2 and 3 are shown since the results of eq. 1 are similar as the results of eq. 2.

First, we focus on the SBH predictions during the first 45 minutes of settling (shown at the top of Figure 2) as this is the typical duration of batch settling experiments. For both hindered settling functions the 1-D model performs well in predicting the initial linear descent of the sludge blanket but the models under predict the sludge blanket height once the curve of the SBH starts bending (corresponding to the onset of compression settling). This under prediction is larger for the hindered settling function of Diehl. From a modelling point of view, this under prediction is as expected. A 1-D model that only accounts for hindered settling will evidently over predict the settling velocity at high concentrations and thus under predict the SBH since it disregards the gradual compressibility the formed particle network may undergo. To include more physical realism a compression function should be added which slows down the settling velocity at high solids concentrations causing the sludge blanket to descend less rapidly.

However, the dataset collected by De Clercq et al. (2005) provides data of up to 6 hours of settling. The full measurements and simulation results for 6 hours of settling are shown at the bottom of Figure 2. These results clearly show that the 1-D model simulations with the exponential function of Takács perform a lot better to describe the overall trend in the experimental data. However, from a physical perspective these predictions do not make sense. The predictions switch from an underprediction of the SBH (at t<2h) to an overprediction of the SBH (at t>2h) whereas we expect the SBH to be underpredicted over the entire experiment since no compression settling is taken into account in these simulations. Hence, using an exponentially decaying function to describe hindered settling does not only account for hindered settling but already lumps in some compressive behaviour (by predicting very low values for v_{hs} at high sludge concentrations - see Figure 1). Although this results in a model that performs relatively well in predicting the general trends, it will hamper further efforts to include more realism by accounting for other phenomena such as compression. Adding any type of compression function will decrease the settling velocity at higher concentrations and thus increase the predicted SBH resulting in somewhat better predictions for t<2h but much worse predictions at t>2h. The power-law function suggested by Diehl (2014) does perform as expected from a hindered settling function and can be combined with a constitutive function for the solids stress to obtain a more advanced 1-D model that accounts for both hindered and compression settling. Whereas the exponential functions are clearly the best choice for simplified settler models that only include hindered settling, they are not suitable if we want to move towards advanced settler models that aim to explicitly account for several phenomena. For the latter models a power-law type function is shown much more appropriate in describing the hindered settling behaviour.

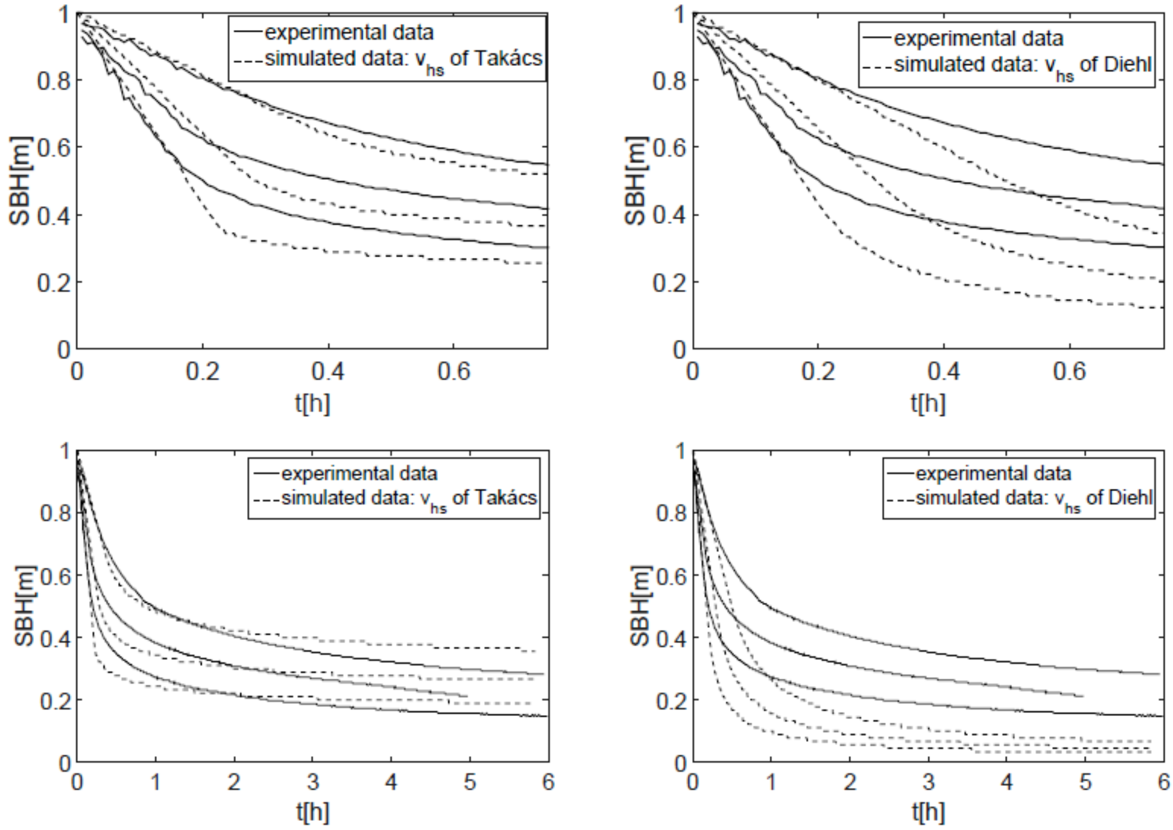

Figure 2: Measured sludge blanket heights (solid line – from De Clercq et al. (2005) and predicted sludge blanket heights (dashed lines) by the Bürger-Diehl model with only hindered settling. Left: hindered settling function of Takács, right: hindered settling function of Diehl (2014).

These findings were validated on the second dataset provided by Locatelli et al. (2014). This dataset provided direct measurements of the settling velocity as well as the concentrations over the entire depth of the settling column. The measured velocity and concentration profiles were compared to 1-D simulation results with the different hindered settling functions. For reasons of brevity the measured and predicted velocity profiles are not shown here. Only the key findings are discussed.

For an experiment at an initial concentration of 3.8 g/l a settling velocity of 0.0124 m/d and a concentration of 32 g/l was measured at the bottom of the settling column after 22 hours of settling. Simulations with the 1-D model with the calibrated hindered settling velocity of Takács predicted a velocity of 0.0019 m/d and a concentration of 21 g/l. These results confirmed that the exponential function of Takács predicts too low settling velocities at high solids concentrations making it unsuitable to combine with a compression function in advanced 1-D models.

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